

UNIVERSITY OF TORONTO
DEPARTMENT OF ECONOMICS

ECONOMICS 381H1F – SUMMER 2014

MANAGERIAL ECONOMICS II: PERSONNEL ECONOMICS

Midterm

Version A

SOLUTIONS

Instructions

The test is 50 minutes long. Non-programmable calculators are allowed. The test consists of four questions, each worth 5 points. Show all your work in the space provided below the question. If you need additional space, you may write on the back of the page.

LAST NAME _____
FIRST NAME _____
STUDENT NUMBER _____

Good luck!

Question 1	Question 2	Question 3	Question 4	Total
/5	/5	/5	/5	/20

1. Consider a relationship where a homeowner contracts an architect to design and oversee the construction of his house. Both the homeowner and the architect are risk averse, with the coefficients of absolute risk aversion of 2 and 4, respectively. The value of the house, once it is built, depends stochastically on the architect's effort according to $q=e+u$, where e is the architect's effort that can be observed and verified by the homeowner and u is a random variable with a mean of 0 and a variance of θ . The architect's cost of effort is given by $c(e)=0.5e^2$. The outside options are 0 for both the homeowner and the architect. Assume that the architect's pay is linear in the value of the house (i.e. $w=a+bq$).

- a. (1 point) What is the expected value of the house if the contract is efficient?
- b. (2 points) What is the optimal risk-sharing contract in this relationship?
- c. (2 points) Is it efficient to form this relationship?

- a. (1 point) The expected value of the house is given by $E[q]=E[e+u]=e$. The efficient contract sets the marginal expected benefit of effort equal to its marginal cost. The marginal expected benefit is equal to $\partial E[q]/\partial e=1$ and the marginal cost is $\partial c(e)/\partial e=e$. Therefore, the efficient level of effort is $e^*=1$, from which it follows that the expected house value is $E[q]=e^*=1$.
- b. (2 points) The optimal risk sharing contract (b) minimizes the sum of the risk premiums for the homeowner and the architect. The risk premium for the homeowner is $0.5s\text{Var}[q-w]$ and the risk premium for the architect is $0.5r\text{Var}[w]$, where s and r are coefficients of absolute risk aversion. Further, $\text{Var}[w] = \text{Var}[a+bq] = b^2\theta$ and $\text{Var}[q-w] = \text{Var}[(1-b)q-a]=(1-b)^2\theta$. Therefore, the risk premium for the homeowner is $0.5(2)(1-b)^2\theta=(1-b)^2\theta$ since $s=2$, and the risk premium for the architect is $0.5(4)b^2\theta=2b^2\theta$ since $r=4$. Therefore, the sum of the risk premiums is $(1-b)^2\theta+2b^2\theta$. The first-order condition for b is equal to $-2(1-b)\theta+4b\theta=0$. Solving for b^* , we have that the optimal risk sharing contract is $b^*=1/3$.
- c. (2 points) It is efficient to form the relationship if the social surplus in the relationship, evaluated at the optimal contract values, is greater than the sum of outside options. The social surplus is equal to $E[V]+E[U]=E[q-w]-RP^P+E[w]-c(e)-RP^A = E[q]-c(e)-RP^P-RP^A$. Now, $E[q]=e^*=1$, $c(e^*)=0.5e^{*2}=0.5$, $RP^P=(1-b^*)^2\theta=(4/9)\theta$, and $RP^A=2b^{*2}\theta=(2/9)\theta$. Therefore, the social surplus is $1-0.5-(4/9)\theta-(2/9)\theta=(1/2)-(2/3)\theta$. This will be a positive number (i.e. greater than the sum of outside options, which is equal to 0), if and only if θ is smaller than or equal to $3/4$. Therefore, if $\theta \leq 3/4$, it is efficient to form the relationship, and if $\theta > 3/4$, it is not efficient to form the relationship.

2. Do you agree with the following statements? Explain why.

- (a) (1 point) Moral hazard arises whenever a principal delegates a task to an agent.
- (b) (1 point) Slavery is not a socially efficient contract.
- (c) (1 point) The expected pay is always higher for riskier jobs.
- (d) (1 point) It is never optimal to tie the agent's pay to the observed outcome.
- (e) (1 point) The contract should never be based on signals unrelated to the agent's effort.

- (a) (1 point) No. It must be the case that the principal cannot observe the agent's action and that the principal and the agent have conflicting goals.
- (b) (1 point) Yes, because an efficient contract requires voluntary participation of both parties. The enslaved party clearly has a better outside option and will never choose to enter into this relationship.
- (c) (1 point) No. If the agent is risk neutral, the expected pay does not depend on the variance of outcome, i.e. risk.
- (d) (1 point) No. When the agent's action cannot be observed by the principal, it is optimal to tie the agent's pay to the observed outcome.
- (e) (1 point) No. The important thing is that the signal is related to the observed outcome.

3. The board of directors of Apple Inc. wishes to hire a new CEO. The board considers a payment method of the form $w=a+bq+cy$, where w is the CEO's pay, q is the revenues for Apple Inc. and y is the revenues of Blackberry Inc. Specifically, the Board knows that that $E[q]=e$, $\text{Var}[q]=2$, $E[y]=0$, $\text{Var}[y]=1$, and $\text{cov}(q,y)=-0.2$, where e is the CEO's effort. The CEO's cost of effort is $0.5e^2$ and his coefficient of risk aversion is 2. On the other hand, the board of Apple Inc. is risk neutral. Assume that both parties have outside options of zero.

- a. (1 point) Write down the expected payoff (i.e. certainty equivalent) for the board of Apple Inc. and the CEO using the information given in the question.
- b. (1 points) What is the maximum expected profit for Apple Inc. if the board can observe and verify the CEO's effort?
- c. (3 points) What is the maximum expected profit for Apple Inc. if the board cannot observe and verify the CEO's effort?

a. (1 point) The expected payoff for the board is $E[V]=E[q]-E[w]$ since the board is risk neutral. Further, $E[q]-E[w]=e-a-be$. The CEO's expected payoff is $E[U]=E[w]-c(e)-RP^A = a+be-0.5e^2-RP^A$. Now, $RP^A=0.5r\text{Var}[w]=0.5(2)\text{Var}[a+bq+cy]=b^2(2)+c^2(1)+2bc(-0.2)=2b^2+c^2-0.4bc$. Therefore, $E[U]=a+be-0.5e^2-2b^2-c^2+0.4bc$.

b. (1 point) The expected profit for the board is $E[q]-E[w]=e-a-be$. If the effort can be observed, the board can choose e such that the expected marginal benefit of effort (1) is equal to its marginal cost (e), or set $e^*=1$. Further, given that the board is risk neutral while the CEO is risk averse, it is optimal that the board fully insures the CEO, i.e. $b^*=0$. Lastly, the board has no need for additional signal of performance since e can be observed. Therefore, $c^*=0$ and $w=a+bq+cy=a$. Now, a must satisfy the CEO's participation constraint, $E[w]-c(e)-RP^A=R=0$. Since $b^*=0=c^*$, $RP^A=0$, and $E[U]=a-0.5(1)^2=0$, or $a^*=0.5$. The expected profit is then $e^*-a^*=1-0.5=0.5$.

c. (3 points) The board maximizes $E[V]$ subject to the CEO's participation and incentive compatibility constraints. The ICC is $\partial E[U]/\partial e=0$, or $\partial(a+be-0.5e^2-2b^2-c^2+0.4bc)/\partial e=0$, from which it follows that $b-e=0$, or $e=b$. The PC is $E[U]=R=0$, or $E[w]=c(e)+RP^A=0.5e^2+2b^2+c^2-0.4bc$. Now, $E[V]=E[q]-E[w]$. Substituting for $E[w]$ from the PC, we have $E[V]=e-0.5e^2-2b^2-c^2+0.4bc$. Substituting for e from the ICC, this becomes $b-0.5b^2-2b^2-c^2+0.4bc$. The first-order conditions for b and c are then, respectively, $1-b-4b+0.4c=0$ and $-2c+0.4b=0$. Solving these two equations for b and c yields $b^*\approx 0.2$ and $c^*\approx 0.04$. From the ICC, this then yields $e^*=b^*=0.2=E[q^*]$. Also, from the PC, we have that $E[w^*]=0.5e^{*2}+2b^{*2}+c^{*2}-0.4b^*c^*\approx 0.1$. Therefore, the expected profit is $E[q^*]-E[w^*]\approx 0.2-0.1=0.1$.

4. To test whether switching from a salary contract to a fee-for-service contract will increase physician productivity, the Ministry of Health collected data on the number of visits by the salary and fee-for-service physicians. These data were then analyzed and the main result of this analysis can be expressed using the following regression model: $E[q] = 15 + 5D$, where q is the number of patient visits and D is an indicator equal to 0 if the physician is paid under a salary contract and 1 if the physician is paid under the fee-for-service contract. The standard error for the coefficient on D was equal to 1.

[Note: a fee-for-service contract is a contract in which the physician is paid a fixed fee for each service that he provides.]

- a. (1 point) What is the average productivity of salary and fee-for-service physicians?
 - b. (2 points) Is the relationship between the type of contract and physician productivity statistically and economically significant?
 - c. (1 point) Is the observed relationship between the type of contract and physician productivity consistent with the evidence in Shearer (2004) on the productivity of tree planters?
 - d. (1 point) If the physicians can choose which type of contract to accept, explain why the observed relationship between the type of contract and physician productivity may not represent a cause-and-effect relationship.
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- a. (1 point) The average productivity is 15 for salary workers and 20 for fee-for-service workers.
 - b. (2 points) The fee-for-service physicians provide on average 5 more visits, and this impact is statistically significant, as the t-statistic is $5/1=5$. Further, the magnitude of the impact is large, $5/15$, or $1/3$, which seems to be economically significant.
 - c. (1 point) The fee-for-service contract is similar to the piece rate contract, and therefore the evidence from this analysis is similar to Shearer (2004), who also finds that the type of contract has a significant impact on productivity. However, the estimate of 33% seems to be higher than the estimate of 20% that Shearer reports from his field experiment.
 - d. (1 point) Physicians who choose different type of contracts may have different productivity even if they were paid according to the same type of contract. Because of this selection effect, the observed impact of 5 visits may not reflect only the treatment effect. In fact, it is even possible that there is no treatment effect, and no causal relationship between physician productivity and the type of payment contract.